

# NOVEL TILTMETER FOR MONITORING ANGLE SHIFT IN INCIDENT WAVES

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## Abstract

*We are presenting here the concept of using the quality factor of a resonator in order to monitor angle shift in an incident signal based on a transmission [1]. As an example, we treat the case where the resonator is a Fabry-Pérot etalon as a tool to monitor the incident angle change of the input waves. We will derive a boundary condition determining when the transmission of an incident wave through the etalon is either more sensitive to the wavelength change of the incident wave or to its incident angle change. That allows us to conclude when the etalon can be used as a tool to monitor the wavelength change of the incident light and when it can monitor its angle change. This concept is not just restricted to the Fabry-Pérot, but is extended to any resonator having both an incident and transmitted light.*

## INTRODUCTION

We are presenting here the concept of using the quality factor of a resonator in order to monitor angle shift in an incident signal based on a transmission. The Q factor or quality factor is a measure of the “quality” of a resonant system. Resonant systems respond to frequencies, angles of incidence, etc. close to their natural frequency, angle of incidence, etc. much more strongly than they respond to other frequencies, angles of incidence, etc. The Q factor indicates the amount of resistance to resonance in a system. Systems with a high Q factor resonate with greater amplitude (at the resonant frequency) than systems with a low Q factor. Damping decreases the Q factor. Resonators have therefore been used to measure and monitor any change in frequency, since a change in frequency would decrease the Q factor of the system. In addition to monitoring a change in frequency, it is desirable in some circumstances to monitor the angle of incidence. However, nothing heretofore devised has used a resonator to monitor this angle. Thus, a continuing need exists for a system that allows a user to monitor the angle of incidence based on a transmission through a resonator. Within some regions of application, the quality factor is much more sensitive to angle shift than it is to frequency shift. Using its quality factor, we propose for the first time to use the resonator to measure any angle shift from the optimum angle of incidence. The resonator is any suitable mechanism or device that allows for the resonant oscillation of an input signal, a non-limiting example of which includes a Fabry-Pérot etalon. Other non-limiting examples include a hollow chamber whose dimensions allow the resonant oscillation of electromagnetic or acoustic waves, and an electrical circuit that combines capacitance and inductance in such a way that a periodic electric oscillation will reach maximum amplitude. Using the resonator, an incident signal is introduced to the resonator. The measured angle is the incident angle of the signal entering the resonator. Each time this angle shifts, it changes the transmitted intensity and rejected intensity. The degree to which the incident

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angle has been changed can be deduced by measuring either the transmitted intensity or the rejected intensity. One example of such embodiment is the one illustrated in Figure 1. However, the novel concept that we introduce here is not just restricted to that structure, but can be expanded to the use of a wide variety of resonators.

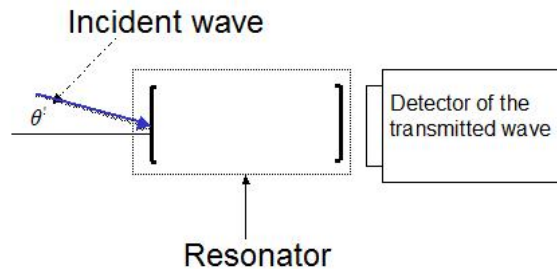


Figure 1. In the following set up, any angle change of the incident beam  $\theta$  results in a change of the intensity transmission of the resonator.

## A NOVEL ANGLE TILTMETER BASED ON THE USE OF A FABRY-PÉROT RESONATOR

For many years the Fabry-Pérot etalon has been used as an efficient tool to monitor wavelength changes of the input light. In addition, we introduce here the concept of using the Fabry-Pérot etalon as a tool to monitor the incident angle change of the input light. We will derive a boundary condition determining when the transmission of an incident light through a Fabry-Pérot etalon is either more sensitive to the wavelength change of the incident light or to its incident angle change. That allows us to conclude when the Fabry-Pérot etalon can be used as a tool to monitor the wavelength change of the incident light and when it can monitor its angle change.

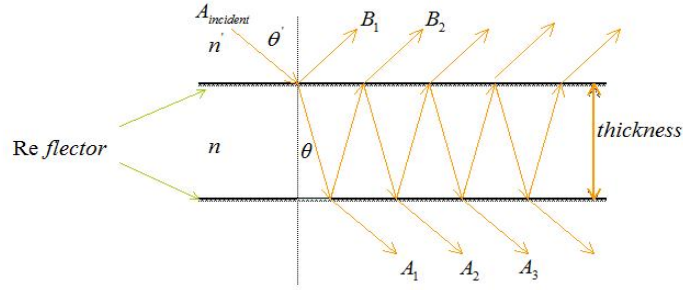


Figure 2. Multiple reflections model for analyzing the Fabry-Pérot etalon; from [2].

A Fabry-Pérot interferometer consisting of a plane-parallel plate of thickness  $l$  and index  $n$  is immersed in a medium of index  $n'$  [1]. Let a plane wave be incident on the etalon at an angle to the normal, as shown in Figure 2. The problem of the transmission (and reflection) of the plane wave through the etalon can be treated by considering the infinite number of partial waves produced by reflections at the two end surfaces. The phase delay between two partial waves, which is attributable to one additional round trip, is given by

$$\delta = \frac{-4.\pi.n.l.\cos(\theta)}{\lambda} \quad (1)$$

where  $\lambda$  is the vacuum wavelength of the incident wave and  $\theta$  is the internal angle of incidence. If the complex amplitude of the incident wave is taken as  $A_i$ , then the partial reflections,  $B_1$ ,  $B_2$  and so forth, are given by:

$$B_1 = rA_i \quad B_2 = tt' r' A_i e^{j\delta} \quad B_3 = tt' r'^3 A_i e^{2j\delta}$$

where  $r$  is the reflection coefficient (ratio of reflected to incident amplitude),  $t$  is the transmission coefficient for waves incident from  $n'$  toward  $n$ , and  $r'$  and  $t'$  are the corresponding quantities for waves incident from  $n$  toward  $n'$ . The complex amplitude of the total reflected wave is:

$$A_r = B_1 + B_2 + B_3 + \dots$$

$$A_r = \{r + tt' r' e^{j\delta} (1 + r'^2 e^{j\delta} + \dots)\} A_i$$

For the transmitted wave,

$$A_1 = tt' A_i, \quad A_2 = tt' r'^2 e^{j\delta} A_i,$$

$$A_3 = tt' r'^4 e^{2j\delta} A_i,$$

Adding up the A terms, we obtain:

$$A_t = A_{it} \left( 1 + r'^2 e^{j\delta} + r'^4 e^{2j\delta} + \dots \right) \quad (2)$$

It can be shown that the Fabry-Pérot etalon has an intensity transmission, which is:

$$Transmission = \frac{(1-R)^2}{(1-R)^2 + 4R \sin\left(\frac{-2\pi n l \cos(\theta)}{\lambda}\right)^2} \quad (3)$$

where R is the fraction of the intensity reflected. Assuming that the geometry and material properties remain constant, equation (1) shows that the transmission is a function of the wavelength of the input signal. For many years, the transmission intensity dependence on the wavelength of the input field has been used to monitor any wavelength change for the input field. Here, we propose the same technique to monitor any angle change of the input field (Figure 2). From equation (1), it is clear that the transmission intensity is also angle-dependent. That can be done since the intensity transmission is also angle-dependent.

For the sake of simplicity, let's denote:

$$T = Transmission$$

$$\frac{\partial T}{\partial \theta} = \frac{8(1-R)^2 \cdot R \cdot \sin\left(\frac{\delta}{2}\right) \frac{\partial R}{\partial \theta} - 4(1-R)^2 \sin\left(\frac{\delta}{2}\right) \frac{\partial R}{\partial \theta} + 4R(1-R)^2 \cos\left(\frac{\delta}{2}\right) \frac{\partial \delta}{\partial \theta}}{(1-R)^2 + 4R \sin\left(\frac{-2\pi n l \cos(\theta)}{\lambda}\right)^2} \quad (4)$$

If the reflection R is angle-independent, meaning  $\frac{\partial R}{\partial \theta} = 0$  within the working range, then

$$\frac{\partial T}{\partial \theta} = -T \cdot \frac{4R \cos\left(\frac{\delta}{2}\right) \frac{\partial \delta}{\partial \theta}}{(1-R)^2 + 4R \sin\left(\frac{\delta}{2}\right)^2} \quad (5)$$

with

$$\frac{\partial \delta}{\partial \theta} = -\frac{4\pi n l \sin(\theta)}{\lambda} \quad (6)$$

Besides,

$$\frac{\partial T}{\partial \lambda} = -T \frac{4R \cos\left(\frac{\delta}{2}\right)}{(1-R)^2 + 4R \sin\left(\frac{\delta}{2}\right)^2} \frac{4\pi n l \cos(\theta)}{\lambda^2} \quad (7)$$

therefore,

$$\frac{\left(\frac{\partial T}{\partial \theta}\right)}{\left(\frac{\partial T}{\partial \lambda}\right)} = \tan(\theta)\lambda \quad (8)$$

The variation of the light intensity transmission through the Fabry-Pérot resonator  $dT$  has two components. One is the contribution  $\frac{dT}{d\theta}$  of an angle change  $d\theta$ ; the other is the contribution  $\frac{dT}{d\lambda}$  of the wavelength change  $d\lambda$ . Therefore, we can express  $dT$  as:

$$dT = \frac{dT}{d\theta} d\theta + \frac{dT}{d\lambda} d\lambda \quad (9)$$

In the case we like to have for a system, which is rather more angle-sensitive than wavelength-sensitive, we shall have:

$$\left| \frac{\partial T}{\partial \theta} d\theta \right| > \left| \frac{\partial T}{\partial \lambda} d\lambda \right| \quad (10)$$

and in the case we would like to monitor the wavelength,

$$\left| \frac{\partial T}{\partial \theta} d\theta \right| < \left| \frac{\partial T}{\partial \lambda} d\lambda \right| \quad (11)$$

Using (8) and under the assumption of small angle change:

$$d\theta \sim \theta \quad (12)$$

$$\frac{\partial T}{\partial \theta} = \tan(\theta)\lambda \frac{\partial T}{\partial \lambda} \sim \theta\lambda \frac{\partial T}{\partial \lambda}$$

So,

$$\left| \frac{\partial T}{\partial \theta} d\theta \right| > \left| \frac{\partial T}{\partial \lambda} d\lambda \right| \Rightarrow \left| \theta^2 \lambda \frac{\partial T}{\partial \lambda} \right| > \left| \frac{\partial T}{\partial \lambda} d\lambda \right|$$

Therefore in satisfying,

$$\theta > \sqrt{\frac{d\lambda}{\lambda}} \quad (13)$$

we will satisfy (10), and in satisfying

$$\theta < \sqrt{\frac{d\lambda}{\lambda}}, \quad (14)$$

we will satisfy (11). From [3], we can monitor wavelength change up to  $d\lambda = 10^{-12} m$  for a central wavelength of  $\lambda = 1.55 \mu m$ . So from (13), with the current technology we are able to detect angle change  $\theta$  as small as few millidegrees, depending on  $\frac{n}{n'}$ .

## REFERENCES

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- [2] A. Yariv, 1997, **Optical Electronics in Modern Communications** (Oxford University Press).
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